



Quiz # 02
Math 101 Section 03 Calculus I
9 October 2024 Wednesday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

- Q-1)** (i) Find m_1 which is the slope of the parabola $y = x^2$ at $P = (s, s^2)$.
- (ii) Find m_2 which is the slope of the parabola $y = x^2 - 4x + 7$ at $Q = (t, t^2 - 4t + 7)$.
- (iii) Find m_3 which is the slope of the line PQ .
- (iv) Setting $m_1 = m_2 = m_3$ solve for s and t if possible.
- (v) If these two parabolas have a common tangent, then write an equation for this common tangent in the form $Ay = Bx + C$, where A , B and C are integers. If no common tangent exists, then explain why.

Grading: $1+1+2+2+4=10$ points

Solution: Grader: `gunes.akbas@bilkent.edu.tr`

(i) $y' = 2x$, so at P we have $m_1 = 2s$.

(ii) $y' = 2x - 4$, so at Q we have $m_2 = 2t - 4$.

(iii) Using the coordinates of the points P and Q we can write $m_3 = \frac{(t^2 - 4t + 7) - (s^2)}{t - s}$.

(iv) From $m_1 = m_2$ we get $s = t - 2$. Substituting this into m_3 and solving $m_3 = m_2$ for t we find $t = \frac{11}{4}$. This in turn gives $s = \frac{3}{4}$.

(v) From (iv) above we found that the line PQ is tangent to both of these parabolas and we saw that $m_1 = m_2 = m_3 = \frac{3}{2}$. Now we can write an equation for this line and after simplifying we get $16y = 24x - 9$.