



Quiz # 02  
Math 101 Section 07 Calculus I  
10 October 2024 Thursday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

Bilkent University

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- Q-1)** (i) Find  $m_1$  which is the slope of the parabola  $y = x^2$  at  $P = (s, s^2)$ .
- (ii) Find  $m_2$  which is the slope of the parabola  $y = x^2 - 4x + 16$  at  $Q = (t, t^2 - 4t + 16)$ .
- (iii) Find  $m_3$  which is the slope of the line  $PQ$ .
- (iv) Setting  $m_1 = m_2 = m_3$  solve for  $s$  and  $t$  if possible.
- (v) If these two parabolas have a common tangent, then write an equation for this common tangent in the form  $Ay = Bx + C$ , where  $A$ ,  $B$  and  $C$  are integers. If no common tangent exists, then explain why.

Grading:  $1+1+2+2+4=10$  points

**Solution:** Grader: `gunes.akbas@bilkent.edu.tr`

(i)  $y' = 2x$ , so at  $P$  we have  $m_1 = 2s$ .

(ii)  $y' = 2x - 4$ , so at  $Q$  we have  $m_2 = 2t - 4$ .

(iii) Using the coordinates of the points  $P$  and  $Q$  we can write  $m_3 = \frac{(t^2 - 4t + 16) - (s^2)}{t - s}$ .

(iv) From  $m_1 = m_2$  we get  $s = t - 2$ . Substituting this into  $m_3$  and solving  $m_3 = m_2$  for  $t$  we find  $t = 5$ . This in turn gives  $s = 3$ .

(v) From (iv) above we found that the line  $PQ$  is tangent to both of these parabolas and we saw that  $m_1 = m_2 = m_3 = 6$ . Now we can write an equation for this line and after simplifying we get  $y = 6x - 9$ .