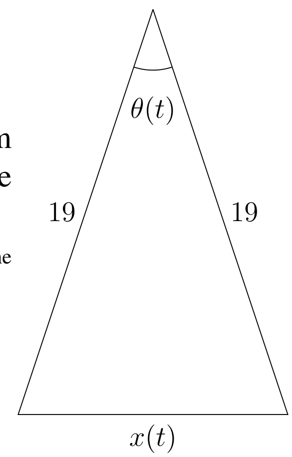
Quiz # 04
Math 101 Section 03 Calculus I
23 October 2024 Wednesday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University



Q-1) In the given isosceles triangle the base length is decreasing at a rate of 3 cm per second and the vertex angle is $\pi/3$ at a given time t_0. Find how fast the vertex angle θ is changing at that time.

Hint: In a triangle with sides of lengths a and b with θ being the angle between them, the third side c is found by the cosine rule which says $c^2 = a^2 + b^2 - 2ab\cos\theta$.

Grading: 10 points

Solution: Grader: `gunes.akbas@bilkent.edu.tr`

From the cosine rule we have

$$\begin{aligned} x(t)^2 &= 2\cdot 19^2 - 2\cdot 19^2\cos\theta(t) \\ &= 2\cdot 19^2(1-\cos\theta(t) \\ &= 2\cdot 19^2\left(2\sin^2\frac{\theta(t)}{2}\right) \\ x(t) &= 2\cdot 19\sin\frac{\theta(t)}{2} \qquad (*) \end{aligned}$$

Putting $t = t_0$ and $\theta(t_0) = \pi/3$ in $(*)$ we find that

$$x(t_0) = 19.$$

You could also immediately conclude that $x(t_0) = 19$ by observing that our triangle will be equilateral when the vertex angle is $\pi/3$.

Differentiating both sides of $(*)$ we find

$$x'(t) = 19\cos\frac{\theta(t)}{2}\,\theta'(t)$$

Here putting in $t = t_0$ with $\theta(t_0) = \pi/3$, $x'(t_0) = -3$ and $x(t_0) = 19$, and solving for $\theta'(t_0)$ we find

$$\theta'(t_0) = \frac{-3}{19}\,\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{19} \approx -0.182\,\text{rad/s} \approx -10.4\,\text{deg/s}.$$

Answer: At the given time the vertex angle is decreasing at a rate of $\dfrac{2\sqrt{3}}{19}$ rad/s.