



Quiz # 04  
Math 101 Section 07 Calculus I  
24 October 2024 Thursday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

Bilkent University

**Q-1)** In the given isosceles triangle the base length is decreasing at a rate of 2 cm per second and the vertex angle is  $\pi/3$  at a given time  $t_0$ . Find how fast the vertex angle  $\theta$  is changing at that time.

Hint: In a triangle with sides of lengths  $a$  and  $b$  with  $\theta$  being the angle between them, the third side  $c$  is found by the cosine rule which says  $c^2 = a^2 + b^2 - 2ab \cos \theta$ .

Grading: 10 points

**Solution:** Grader: `gunes.akbas@bilkent.edu.tr`

From the cosine rule we have

$$\begin{aligned}x(t)^2 &= 2 \cdot 17^2 - 2 \cdot 17^2 \cos \theta(t) \\&= 2 \cdot 17^2 (1 - \cos \theta(t)) \\&= 2 \cdot 17^2 \left( 2 \sin^2 \frac{\theta(t)}{2} \right) \\x(t) &= 2 \cdot 17 \sin \frac{\theta(t)}{2} \quad (*)\end{aligned}$$

Putting  $t = t_0$  and  $\theta(t_0) = \pi/3$  in (\*) we find that

$$x(t_0) = 17.$$

You could also immediately conclude that  $x(t_0) = 17$  by observing that our triangle will be equilateral when the vertex angle is  $\pi/3$ .

Differentiating both sides of (\*) we find

$$x'(t) = 17 \cos \frac{\theta(t)}{2} \theta'(t)$$

Here putting in  $t = t_0$  with  $\theta(t_0) = \pi/3$ ,  $x'(t_0) = -2$  and  $x(t_0) = 17$ , and solving for  $\theta'(t_0)$  we find

$$\theta'(t_0) = \frac{-2}{17} \frac{2}{\sqrt{3}} \approx -0.135 \text{ rad/s} \approx -7.7 \text{ deg/s}.$$

**Answer:** At the given time the vertex angle is decreasing at a rate of  $\frac{4}{17\sqrt{3}}$  rad/s.

