



Quiz # 06  
Math 101 Section 03 Calculus I  
20 November 2024 Wednesday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

Bilkent University

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**Q-1)**

(i) Interpret  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{(n-k)^{1/2}(n+k)^{1/2}}{n^2}$  as a Riemann sum for a definite integral and evaluate the integral, thus finding the value of this limit.

(ii) Evaluate  $\int_0^{\pi/2} \cos \theta \, d\theta$ .

(iii) Evaluate  $\int_0^{\pi/2} \cos^2 \theta \, d\theta$ .

(iv) Let  $F(\theta) = (\sin \theta \cos^2 \theta + 2 \sin \theta)/3$ . Simplify  $F'(\theta)$  as much as possible and use it to evaluate  $\int_0^{\pi/2} \cos^3 \theta \, d\theta$ .

Grading: 4+1+1+4=10 points

**Solution:** Grader: gunes.akbas@bilkent.edu.tr

**(i)**

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{(n-k)^{1/2}(n+k)^{1/2}}{n^2} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \sqrt{1 - \left(\frac{k}{n}\right)^2}$$
$$\int_0^1 \sqrt{1-x^2} \, dx = \frac{\pi}{4},$$

since the integral gives the area of the quarter of the unit disk in the first quadrant.

**(ii)**  $\int_0^{\pi/2} \cos \theta \, d\theta = \left( \sin \theta \Big|_0^{\pi/2} \right) = 1$ , since  $(\sin \theta)' = \cos \theta$ .

**(iii)**  $\int_0^{\pi/2} \cos^2 \theta \, d\theta = \int_0^{\pi/2} \frac{1 + \cos 2\theta}{2} \, d\theta = \left( \frac{\theta}{2} + \frac{1}{4} \sin 2\theta \Big|_0^{\pi/2} \right) = \frac{\pi}{4}$ .

**(iv)** Here  $F'(\theta) = \cos^3 \theta$ . Hence  $\int_0^{\pi/2} \cos^3 \theta \, d\theta = \left( F(\theta) \Big|_0^{\pi/2} \right) = \frac{2}{3}$ .