Quiz # 06
Math 101 Section 07 Calculus I
21 November 2024 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Q-1)

(i) Interpret $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{(n-k)^{1/2}(n+k)^{1/2}}{n^2}$ as a Riemann sum for a definite integral and evaluate the integral, thus finding the value of this limit.

(ii) Evaluate $\int_0^{\pi/2}\cos\theta\ d\theta$.

(iii) Evaluate $\int_0^{\pi/2}\cos^2\theta\ d\theta$.

(iv) Let $F(\theta) = (\sin\theta\cos^2\theta + 2\sin\theta)/3$. Simplify $F'(\theta)$ as much as possible and use it to evaluate $\int_0^{\pi/2}\cos^3\theta\ d\theta$.

Grading: 4+1+1+4=10 points

Solution: Grader: `gunes.akbas@bilkent.edu.tr`

(i)

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{(n-k)^{1/2}(n+k)^{1/2}}{n^2} = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sqrt{1-\left(\frac{k}{n}\right)^2}$$
$$\int_0^1\sqrt{1-x^2}\ dx = \frac{\pi}{4},$$

since the integral gives the area of the quarter of the unit disk in the first quadrant.

(ii) $\int_0^{\pi/2}\cos\theta\ d\theta = \left(\sin\theta\Big|_0^{\pi/2}\right) = 1$, since $(\sin\theta)' = \cos\theta$.

(iii) $\int_0^{\pi/2}\cos^2\theta\ d\theta = \int_0^{\pi/2}\frac{1+\cos 2\theta}{2}\ d\theta = \left(\frac{\theta}{2}+\frac{1}{4}\sin 2\theta\Big|_0^{\pi/2}\right) = \frac{\pi}{4}$.

(iv) Here $F'(\theta) = \cos^3\theta$. Hence $\int_0^{\pi/2}\cos^3\theta\ d\theta = \left(F(\theta)\Big|_0^{\pi/2}\right) = \frac{2}{3}$.