Bilkent University

Quiz # 08
Math 101 Section 03 Calculus I
4 December 2024 Wednesday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Let R be the region bounded by the curves $y = x^2 - 5x + 6$ and $y = 6 - x$. Let A denote the volume obtained by revolving the region R around x-axis, and B denote the volume obtained by revolving the region R around y-axis.

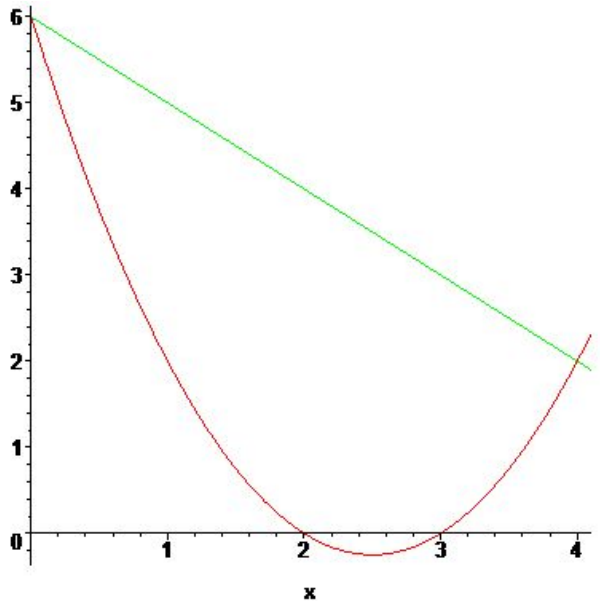
(i) Set up an integral expression that calculates the volume A. *Do not evaluate the integral.*

(ii) Set up an integral expression that calculates the volume B. *Do not evaluate the integral.*

(iii) Now calculate B explicitly.

Grading: 4+3+3=10 points

Solution: Grader: gunes.akbas@bilkent.edu.tr



Solving $x^2 - 5x + 6 = 6 - x$ we see that the curves intersect at $(0, 6)$ and $(4, 2)$. We also notice that the curve $y = x^2 - 5x + 6$ intersects x axis at $x = 2$ and $x = 3$. Therefore the portion of the parabola under the x-axis between $x = 2$ and $x = 3$ will not contribute to the volume A since the part above x-axis already generates the volume contributed by the part below x-axis.

$$A = \pi \int_0^2 \left[(6-x)^2 - (x^2 - 5x + 6)^2\right]\, dx + \pi \int_2^3 \left[(6-x)^2\right]\, dx + \pi \int_3^4 \left[(6-x)^2 - (x^2 - 5x + 6)^2\right]\, dx$$

$$B = 2\pi \int_0^4 x\left[(6-x) - (x^2 - 5x + 6)\right]\, dx$$

$$B = \frac{128}{3}\pi. \quad \textit{(In case you are wondering } A = \frac{1537}{30}\pi)$$