Bilkent University

Quiz # 08
Math 101 Section 07 Calculus I
5 December 2024 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Let R_A be the region bounded by the curves $x^2+4x+y+3=0$ and $x+y+3=0$. Let R_B be the region bounded by the curves $x^2+4x+y+3=0$, $x+y+3=0$ and $x+1=0$ for $-1\le x\le 0$. Let A denote the volume obtained by revolving the region R_A around y-axis, and B denote the volume obtained by revolving the region R_B around x-axis.

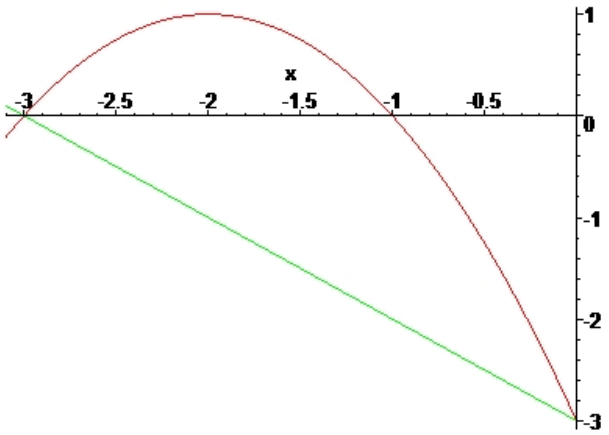
(i) Calculate A.

(ii) Calculate B.

Hint: $(x^2+4x+3)^2=x^4+8x^3+22x^2+24x+9$.

Grading: 5+5=10 points

Solution: Grader: gunes.akbas@bilkent.edu.tr

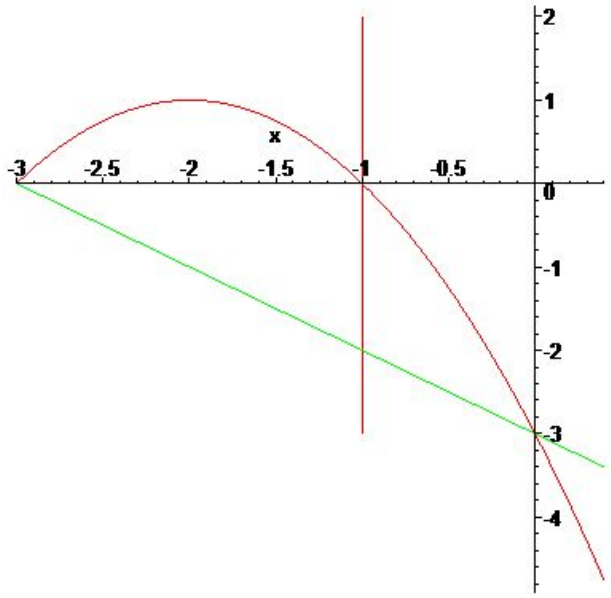


Solving $x^2+4x+3=x+3$ we see that the curves intersect at $(-3,0)$ and $(0,-3)$. We also notice that the curve $x^2+4x+y+3=0$ intersects x axis at $x=-3$ and $x=-1$.

(i) For the calculation of A we use the cylindrical shell method. Note here that the distance from y-axis is $-x>0$. The curves we are dealing with are $y=-x^2-4x-3$ and $y=-x-3$.

$$\begin{aligned} A &= 2\pi\int_{-3}^{0}(-x)\left[(-x^2-4x-3)-(-x-3)\right]\,dx \\ &= 2\pi\int_{-3}^{0}(x^3+3x^2)\,dx = 2\pi\left(\frac{x^4}{4}+x^3\Big|_{-3}^{0}\right)=\frac{27}{2}\,\pi. \end{aligned}$$

(ii) Here we use the disc method.



$$
\begin{aligned}
B &= \pi \int_{-1}^{0} \left[(-x-3)^2 - (-x^2-4x-3)^2\right]\, dx \\
&= \pi \int_{-1}^{0} \left[-x^4 - 8x^3 - 21x^2 - 18x\right]\, dx \\
&\pi \left(-\frac{x^5}{5} - 2x^4 - 7x^3 - 9x^2 \Big|_{-1}^{0} \right) \\
&= \frac{19}{5}\,\pi.
\end{aligned}
$$