



Quiz # 09  
Math 101 Section 03 Calculus I  
11 December 2024 Wednesday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

Bilkent University

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**Q-1)**

(i) If  $f(x) = x^\pi + \pi^x + \log_\pi x + e^\pi$ , then find  $\int f(x) dx$ .

(ii) If  $f(x) = \log_x \pi$ , then find  $f'(e)$ .

(iii) If  $f(x) = (\cos x + x^2)^{\sin x + 3x}$ , then find  $f'(0)$ .

Hint:  $(x \ln x - x)' = \ln x$ .

Grading: 4+4+2=10 points

**Solution:** Grader: `gunes.akbas@bilkent.edu.tr`

(i) Here we first note the following:

$$\pi^x = y \iff x \ln \pi = \ln y \iff y = e^{x \ln \pi}$$

$$\log_\pi x = y \iff x = \pi^y \iff \ln x = y \ln \pi \iff y = \frac{\ln x}{\ln \pi}$$

Then  $f(x) = x^\pi + e^{x \ln \pi} + \frac{\ln x}{\ln \pi} + e^\pi$  and hence

$$\int f(x) dx = \frac{x^{\pi+1}}{\pi+1} + \frac{\pi^x}{\ln \pi} + \frac{x \ln x - x}{\ln \pi} + e^\pi x + C.$$

(ii) Here we again have to untangle the meaning of  $f(x)$ .

$$f(x) = \log_x \pi \iff \pi = x^{f(x)} \iff \ln \pi = f(x) \ln x \iff f(x) = \frac{\ln \pi}{\ln x}.$$

Now we can easily write

$$f'(x) = \left( \frac{\ln \pi}{\ln x} \right)' = -\frac{\ln \pi}{x(\ln x)^2},$$

and hence

$$f'(e) = -\frac{\ln \pi}{e}.$$

(iii) Here the calculations are straightforward.

$$f(x) = (\cos x + x^2)^{\sin x + 3x}$$

$$= \exp[(\sin x + 3x) \ln(\cos x + x^2)]$$

$$f'(x) = f(x)[(\cos x + 3) \ln(\cos x + x^2) + (\sin x + 3x) \cdot \frac{-\sin x + 2x}{\cos x + x^2}]$$

$$f'(0) = 0.$$