Quiz # 09
Math 101 Section 07 Calculus I
12 December 2024 Thursday
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Solution Key

Bilkent University

Q-1)

(i) If $f(x) = x^\pi + \pi^x + \log_\pi x + e^\pi$, then find $\int f(x)\,dx$.

(ii) If $f(x) = \log_x \pi$, then find $f'(e)$.

(iii) If $f(x) = (\cos x + x^2)^{\sin x + 3x}$, then find $f'(0)$.

Hint: $(x \ln x - x)' = \ln x$.

Grading: 4+4+2=10 points

Solution: Grader: `gunes.akbas@bilkent.edu.tr`

(i) Here we first note the following:
$\pi^x = y \iff x \ln \pi = \ln y \iff y = e^{x \ln \pi}$
$\log_\pi x = y \iff x = \pi^y \iff \ln x = y \ln \pi \iff y = \frac{\ln x}{\ln \pi}$

Then $f(x) = x^\pi + e^{x \ln \pi} + \frac{\ln x}{\ln \pi} + e^\pi$ and hence

$$\int f(x)\,dx = \frac{x^{\pi+1}}{\pi+1} + \frac{\pi^x}{\ln \pi} + \frac{x \ln x - x}{\ln \pi} + e^\pi x + C.$$

(ii) Here we again have to untangle the meaning of $f(x)$.
$f(x) = \log_x \pi \iff \pi = x^{f(x)} \iff \ln \pi = f(x) \ln x \iff f(x) = \frac{\ln \pi}{\ln x}$.

Now we can easily write

$$f'(x) = \left(\frac{\ln \pi}{\ln x}\right)' = -\frac{\ln \pi}{x(\ln x)^2},$$

and hence

$$f'(e) = -\frac{\ln \pi}{e}.$$

(ii) Here the calculations are straightforward.

$$\begin{aligned} f(x) &= (\cos x + x^2)^{\sin x + 3x} \\ &= \exp[(\sin x + 3x)\ln(\cos x + x^2)] \\ f'(x) &= f(x)[(\cos x + 3)\ln(\cos x + x^2) + (\sin x + 3x)\cdot \frac{-\sin x + 2x}{\cos x + x^2}] \\ f'(0) &= 0. \end{aligned}$$