Bilkent University

Quiz # 02
Math 101 Section 01 Calculus I
7 October 2025, Tuesday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the function

$$\phi(x) = \begin{cases} x^3 + 2x & x \leq 3 \\ ax + b & x > 3 \end{cases}$$

where a and b are constants to be determined.

(i) Find all a and b for which $\phi(x)$ is continuous at $x = 3$.

(ii) Find all a and b for which $\phi(x)$ is differentiable at $x = 3$.

Grading: 5+5=10 points
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Solution:

(i) For ease of notation let $f(x) = x^3 + 2x$ and $g(x) = ax + b$.

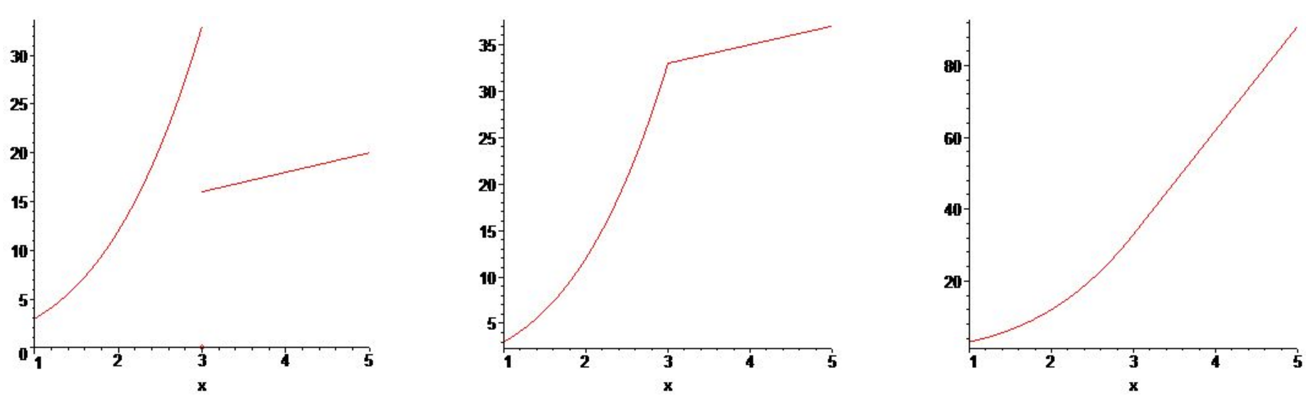
For continuity at $x = 3$ we need to have $f(3) = g(3)$. This gives $33 = 3a + b$. Thus for all such a and b, the function $\phi(x)$ is continuous at $x = 3$.

(ii) Since ϕ cannot be differentiable at $x = 3$ if it is not already continuous there, we must first have $33 = 3a + b$.

Next we need the slopes of both f and g agree at $x = 3$ so that ϕ will be differentiable there.

Note that $f'(x) = 3x^2 + 2$, $f'(3) = 29$, $g'(x) = a$, $g'(3) = a$. Since we need $f'(3) = g'(3)$, we must have $a = 29$. This then together with $3a + b = 33$ gives $b = -54$.

Conclusion: $\phi(x)$ is differentiable at $x = 3$ only for the values $a = 29$ and $b = -54$.



In the first graph $a = 2$, $b = 10$ and ϕ is discontinuous. In the second graph $a = 2$, $b = 27$ and ϕ is continuous. In the third graph $a = 29$, $b = -54$ and ϕ is differentiable.