1) Plot the graph of \( r = 1 + 2 \cos(\theta) \), for \( 0 \leq \theta \leq 2\pi \).

Solution-1) The graph is symmetric about the \( x \)-axis.
When \( 0 \leq \theta \leq \pi/2 \),\( \cos \theta \) decreases from 1 to 0, so \( r \) decreases from 3 to 1.
When \( \pi/2 \leq \theta \leq 2\pi/3 \), \( \cos \theta \) decreases from 0 to \(-1/2\), so \( r \) decreases from 1 to 0.
When \( 2\pi/3 \leq \theta \leq \pi \), \( \cos \theta \) decreases from \(-1/2\) to \(-1\), so \( r \) decreases from 0 to \(-1\).
Now flipping the resulting curve about the \( x \)-axis gives the full graph.
2) Let \( \omega \) be a differentiable function of \( x \) and \( y \), and let \( x = f(t) \) and \( y = g(t) \), where \( f \) and \( g \) are differentiable functions of \( t \). Using the table below, calculate \( \frac{d^2 \omega}{dt^2}(0) \).

<table>
<thead>
<tr>
<th></th>
<th>( f(0) = 1 )</th>
<th>( f(1) = 0 )</th>
<th>( g(0) = 1 )</th>
<th>( g(1) = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f'(0) = 2 )</td>
<td>( f'(1) = 3 )</td>
<td>( g'(0) = 4 )</td>
<td>( g'(1) = 6 )</td>
</tr>
<tr>
<td></td>
<td>( f''(0) = -3 )</td>
<td>( f''(1) = -2 )</td>
<td>( g''(0) = 10 )</td>
<td>( g''(1) = 11 )</td>
</tr>
</tbody>
</table>

\[
\frac{\partial \omega}{\partial x}(0,0) = 31 \quad \frac{\partial \omega}{\partial x}(1,1) = 13 \quad \frac{\partial \omega}{\partial y}(0,0) = 18 \quad \frac{\partial \omega}{\partial y}(1,1) = 8 \quad \frac{\partial^2 \omega}{\partial x \partial y}(0,0) = 3
\]

\[
\frac{\partial^2 \omega}{\partial x^2}(0,0) = 6 \quad \frac{\partial^2 \omega}{\partial x^2}(1,1) = 7 \quad \frac{\partial^2 \omega}{\partial y^2}(0,0) = 8 \quad \frac{\partial^2 \omega}{\partial y^2}(1,1) = -6 \quad \frac{\partial^2 \omega}{\partial x \partial y}(1,1) = 5
\]

**Solution-2**

\[
\frac{d\omega}{dt}(0) = \frac{\partial \omega}{\partial x}(1,1) \cdot f'(0) + \frac{\partial \omega}{\partial y}(1,1) \cdot g'(0)
\]

\[
\frac{d^2 \omega}{dt^2}(0) = \left( \frac{\partial^2 \omega}{\partial x^2}(1,1) \cdot f'(0) + \frac{\partial^2 \omega}{\partial y \partial x}(1,1) \cdot g'(0) \right) \cdot f'(0) + \frac{\partial \omega}{\partial x}(1,1) \cdot f''(0)
\]

\[
+ \left( \frac{\partial^2 \omega}{\partial x \partial y}(1,1) \cdot f'(0) + \frac{\partial^2 \omega}{\partial y^2}(1,1) \cdot g'(0) \right) \cdot g'(0) + \frac{\partial \omega}{\partial y}(1,1) \cdot g''(0)
\]

\[
= ((7)(2) + (5)(4))(2) + (13)(-3)
\]

\[
+ ((5)(2) + (-6)(4))(4) + (8)(10)
\]

\[
= 53.
\]
3) Let \( E \) be the tangent plane to the surface \( 3x^2 + 4y^2 - 2z^2 = 1 \) at the point \((1, 2, 3)\). Let 
\[ \omega = x^2 + 8xy + 8y^3 + z^5, \]
subject to the condition that \((x, y, z) \in E\).

Calculate \( \left( \frac{\partial \omega}{\partial x} \right)_z \) at the point \((x, z) = (1, -1)\).

**Solution-3)** The surface is given by \( f(x, y, z) = 3x^2 + 4y^2 - 2z^2 - 1 = 0 \). The gradient of \( f \) is 
\( \nabla f = (6x, 8y, -4z) \). Evaluating at the point \((1, 2, 3)\) gives 
\( \nabla f(1, 2, 3) = (6, 16, -12) \) which is the normal vector of the plane \( E \). Thus the equation of \( E \) is 
\( g(x, y, z) = 6 \cdot (x - 1) + 16 \cdot (y - 2) - 12 \cdot (z - 3) = 0 \), or 
\( g(x, y, z) = 3x + 8y - 6z - 1 = 0 \).

Now differentiate \( \omega \) with respect to \( x \) keeping in mind that \( z \) is free but \( y \) is dependent:

\[ \left( \frac{\partial \omega}{\partial x} \right)_z (x, y, z) = 2x + 8y + 8x \frac{\partial y}{\partial x} + 24y^2 \frac{\partial y}{\partial x}. \]

From the restraint \( g = 0 \) we get by differentiating both sides with respect to \( x \), 
\[ 3 + 8 \frac{\partial y}{\partial x} = 0, \]
or 
\[ \frac{\partial y}{\partial x} = -\frac{3}{8}. \]

From \( g(1, y, -1) = 0 \), we find \( y = -1 \). Finally substituting in these values we get

\[ \left( \frac{\partial \omega}{\partial x} \right)_z (1, -1, -1) = 2 - 8 + 8(-3/8) + 24(-3/8) = -18. \]

4) What is the largest value that the function \( f(x, y) = 6xy - 4x^3 - 3y^2 \) can take?

**Solution-4)** \( f_x = 6y - 12x^2, \ f_y = 6x - 6y \). From \( f_x = f_y = 0 \) we find that the critical points are \((0, 0)\) and \((1/2, 1/2)\).

\[ f_{xx} = -24x, \ f_{xy} = 6, \ f_{yy} = -6. \]

\[ \Delta = f_{xx} f_{yy} - (f_{xy})^2. \]

At \((0, 0)\), \( \Delta(0, 0) = -36 < 0 \), so \((0, 0)\) is a saddle point.

At \((1/2, 1/2)\), \( \Delta(1/2, 1/2) = 36 > 0 \) and \( f_{xx}(1/2, 1/2) = -12 < 0 \), so \((1/2, 1/2)\) is a local maximum point. The value of \( f \) at this local maximum point is \( f(1/2, 1/2) = 1/4 \).

However, the function has neither global maximum nor global minimum values as can be seen by checking the limits 
\[ \lim_{x \to \infty} f(x, 0) = -\infty \] and \( \lim_{x \to -\infty} f(x, 0) = \infty. \)

5) Find the minimum and maximum values of \( f(x, y) = x + 2y + 3 \) subject to the condition that \( 4x^2 + 5y^2 = 84/5 \).

**Solution-5)** Let \( g(x, y) = 4x^2 + 5y^2 - 84/5. \) \( \nabla f = \lambda \nabla g \) gives \((1, 2) = \lambda(8x, 10y)\), or \( x = 1/(8\lambda), \ y = 1/(5\lambda) \). Using the constraint \( g(1/(8\lambda), 1/(5\lambda)) = 0 \), we find that \( x = \pm1 \) and \( y = \pm8/5. \)

Then the maximum value of \( f \) is \( f(1, 8/5) = 36/5 \), and the minimum value is \( f(-1, -8/5) = -6/5. \)