Math 102 Calculus II
Quiz-1
Solutions

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Sec01-1) Evaluate $\int \frac{\ln x}{x} dx$, using integration by parts technique.

Solution: Let $u = \ln x$, $dv = \frac{dx}{x}$. Then $du = \frac{dx}{x}$, $v = \ln x$, and we have

$$\int \frac{\ln x}{x} dx = (\ln x)^2 - \int \frac{\ln x}{x} dx.$$

From here it follows that

$$\int \frac{\ln x}{x} dx = \frac{1}{2}(\ln x)^2 + C.$$

Sec01-2) Evaluate $\int \frac{x^2 + x - 1}{x(x^2+1)}\, dx$.

Solution: First by using partial fractions technique we write

$$\begin{aligned} \frac{x^2 + x - 1}{x(x^2+1)} &= \frac{A}{x} + \frac{Bx + C}{x^2 + 1} \\ &= \frac{(A+B)x^2 + Cx + A}{x(x^2+1)} \\ &= -\frac{1}{x} + \frac{2x}{x^2+1} + \frac{1}{x^2+1}. \end{aligned}$$

It then follows that

$$\begin{aligned} \int \frac{x^2 + x - 1}{x(x^2+1)}\, dx &= -\int \frac{1}{x}\, dx + \int \frac{2x}{x^2+1}\, dx + \int \frac{1}{x^2+1}\, dx \\ &= -\ln|x| + \ln(x^2+1) + \arctan x + C. \end{aligned}$$

Sec02-1) Evaluate $\int x^2 \cos x \, dx$, using integration by parts technique.

Solution: Let $u = x^2$, $dv = \cos x \, dx$. Then $du = 2x \, dx$, $v = \sin x$, and we have

$$\int x^2 \cos x \, dx = x^2 \sin x - 2 \int x \sin x \, dx.$$

Now to evaluate the second integral we again use integration by parts technique and let $u = x$, $dv = \sin x \, dx$. Then $du = dx$, $v = -\cos x$, and we have

$$\int x \sin x \, dx = -x \cos x + \int \cos x \, dx. = -x \cos x + \sin x + C$$

Putting these together we get

$$\int x^2 \cos x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C.$$

Sec02-2) Evaluate $\int \dfrac{x^2 + x + 2}{x(x+1)^2} \, dx$.

Solution: First by using partial fractions technique we write

$$\begin{aligned} \frac{x^2 + x + 2}{x(x+1)^2} &= \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \\ &= \frac{(A+B)x^2 + (2A + B + C)x + A}{x(x+1)^2} \\ &= \frac{2}{x} - \frac{1}{x+1} - \frac{2}{(x+1)^2}. \end{aligned}$$

It then follows that

$$\begin{aligned} \int \frac{x^2 + x + 2}{x(x+1)^2} \, dx &= \int \frac{2}{x} \, dx - \int \frac{1}{x+1} \, dx - \int \frac{2}{(x+1)^2} \, dx \\ &= 2 \ln |x| - \ln |x+1| + \frac{2}{x+1} + C. \end{aligned}$$