Math 102 Calculus II
Quiz-3
Solutions

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Sec01) Find the equation of the tangent plane and the normal line at the point $P_0 = (1, 2, 3)$ of the level surface $f(x, y, z) = f(1, 2, 3)$ where $f(x, y, z) = x^2 + 7xy^3 + z^3 + \arcsin(x/y) + yz + 1778$.

Solution:

$$\nabla f = (2x + 7y^3 + \frac{1}{\sqrt{1-(x/y)^2}}(1/z), 21xy^2 + z, \frac{1}{\sqrt{1-(x/y)^2}}(-x/y^2) + y),$$

$$\nabla f(P_0) = (58 + \frac{1}{2\sqrt{2}}, 87, \frac{-1}{6\sqrt{2}} + 2).$$

The tangent plane is given by the equation:

$$(58 + \frac{1}{2\sqrt{2}})(x-1) + 87(y-2) + (\frac{-1}{6\sqrt{2}} + 2)(z-3) = 0.$$

The normal line is given by the parametric equations, for $t \in \mathbb{R}$:
$x = 1 + (58 + \frac{1}{2\sqrt{2}})t$,
$y = 2 + 87t$,
$z = 3 + (\frac{-1}{6\sqrt{2}} + 2)t$.

Sec02) Find the equation of the tangent plane and the normal line at the point $P_0 = (1/2, 2, 3)$ of the level surface $f(x, y, z) = f(1/2, 2, 3)$ where $f(x, y, z) = x^2 + xy^3 + z^3 + \arccos x^2 + 778$.

Solution:

$$\nabla f = (2x + y^3 - \frac{1}{\sqrt{1-x^2}}, 3y^2x, 3z^2),$$

$$\nabla f(P_0) = (9 - \frac{4}{\sqrt{15}}, 6, 27).$$

The tangent plane is given by the equation:

$$(9 - \frac{4}{\sqrt{15}})(x - 1/2) + 6(y-2) + 27(z-3).$$

The normal line is given by the parametric equations, for $t \in \mathbb{R}$:
$x = 1/2 + (9 - \frac{4}{\sqrt{15}})t$,
$y = 2 + 6t$,
$z = 3 + 27t$.
