Math 102 Calculus II – Quiz-4
Solutions

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Sec01) Let R be the region that lies inside the circle $x^2+y^2=25$ and above the line $y=x-1$. Write $\iint_R dA$ as an iterated integral in two different ways.

Solution: The line and the circle intersect at the points $(-3,-4)$ and $(4,3)$. Using this we have

$$\iint_R dx\,dy = \int_{-4}^{3}\int_{-\sqrt{25-y^2}}^{y+1} dx\,dy + \int_{3}^{5}\int_{-\sqrt{25-y^2}}^{\sqrt{25-y^2}} dx\,dy$$

and

$$\iint_R dy\,dx = \int_{-5}^{-3}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} dy\,dx + \int_{-3}^{4}\int_{x-1}^{\sqrt{25-x^2}} dy\,dx.$$

Each of these integrals is equal to

$$\frac{25}{2}\,\pi + 7/2 - \frac{25}{2}\,\arcsin(3/5) + \frac{25}{2}\,\arcsin(4/5) = 46.31733454.$$

Sec02) Let R be the region inside the parabola $y^2=25-x$, bounded by the line $y=x+17$ and the line $x=9$. Write $\iint_R dA$ as an iterated integral in two different ways.

Solution: The line $y=x+17$ intersects the parabola at the points $(-24,-7)$ and $(-11,6)$. The line $x=9$ intersects the parabola at the points $(9,-4)$ and $(9,4)$. Using this we have

$$\iint_R dx\,dy = \int_{-7}^{-4}\int_{y-17}^{25-y^2} dx\,dy + \int_{-4}^{4}\int_{y-17}^{9} dx\,dy + \int_{4}^{6}\int_{y-17}^{25-y^2} dx\,dy$$

and

$$\iint_R dy\,dx = \int_{-24}^{-11}\int_{-\sqrt{25-x}}^{x+17} dy\,dx + \int_{-11}^{9}\int_{-\sqrt{25-x}}^{\sqrt{25-x}} dy\,dx.$$

Each of these integrals is equal to $\dfrac{1685}{6} = 280.8333333.$
