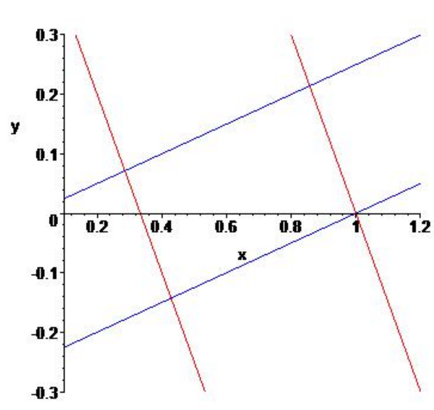
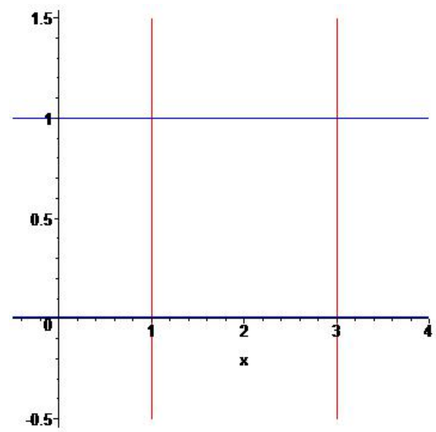
NAME: STUDENT NO:

Q-4) Find the area of the region R in the first quadrant bounded by the lines $3x + 2y = 1$, $3x + 2y = 3$, $x - 4y = 0$ and $x - 4y = 1$.

Hint: Use the transformation $u = 3x + 2y$, $v = x - 4y$ and the associated substitution.

Solution:





The given region is the one on the left. The given transformation takes it to the region on the right.

The Jacobian of this transformation is

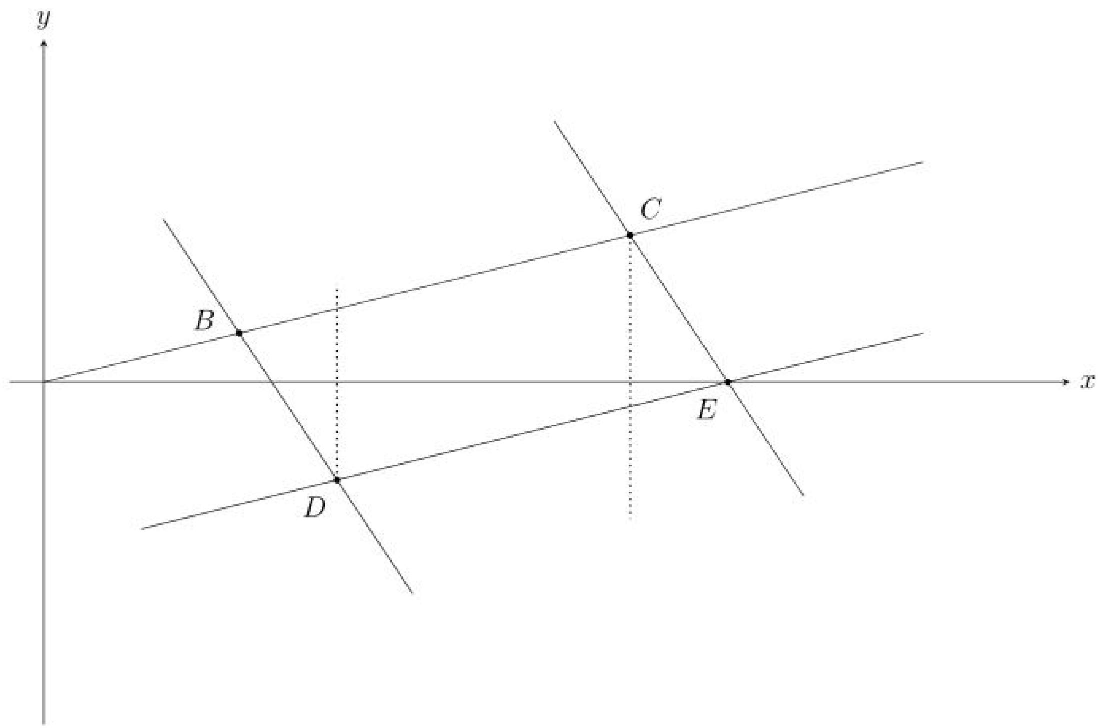
$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 1 & -4 \end{vmatrix} = -14.$$

We need the absolute value of the Jacobian of the reverse transformation, from uv-plane to xy-plane. This value is $\frac{1}{14}$.

The area of the region in the uv-plane is 2.

Therefore the area of the region in the xy-plane is $\frac{1}{7}$.

Another Solution: *see next page*



The coordinates of the above points are as follows.

$$B = \left(\frac{2}{7}, \frac{1}{14}\right), \; C = \left(\frac{6}{7}, \frac{3}{14}\right), \; D = \left(\frac{3}{7}, -\frac{1}{7}\right), \; E = (1, 0),$$

The equation of the line BC is $y = \dfrac{x}{4}$.

The equation of the line DE is $y = \dfrac{x}{4} - \dfrac{1}{4}$.

The equation of the line BD is $y = \dfrac{1}{2} - \dfrac{3}{2}x$.

The equation of the line CE is $y = \dfrac{3}{2} - \dfrac{3}{2}x$.

The area of the region can now be directly calculated as follows.

$$\begin{aligned} Area &= \int_{2/7}^{3/7} \left[\left(\frac{x}{4}\right) - \left(\frac{1}{2} - \frac{3}{2}x\right)\right] dx + \int_{3/7}^{6/7} \left[\left(\frac{x}{4}\right) - \left(\frac{x}{4} - \frac{1}{4}\right)\right] dx \\ &\quad + \int_{6/7}^{1} \left[\left(\frac{3}{2} - \frac{3}{2}x\right) - \left(\frac{x}{4} - \frac{1}{4}\right)\right] dx \\ &= \frac{1}{56} + \frac{3}{28} + \frac{1}{56} \\ &= \frac{1}{7}. \end{aligned}$$