

Q-1) Check the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{3 \cdot 6 \cdot 9 \cdots (3n)}{\pi^n n!}.$$

Solution: Use ratio test, $\frac{a_{n+1}}{a_n} = \frac{3n+3}{\pi(n+1)} \rightarrow \frac{3}{\pi} < 1$ as $n \rightarrow \infty$, to conclude that the series converges.

In fact, $a_n = \left(\frac{3}{\pi}\right)^n$, so the series is a geometric series which starts with $n = 1$. The sum is then found to be $\frac{3}{\pi} \frac{1}{1 - 3/\pi} = \frac{3}{\pi - 3}$.