NAME: STUDENT NO:

**Q-2)** Let $S$ be the cap cut off from the hemisphere $x^2+y^2+z^2=R^2$, $z \geq 0$ by the cylindrical surface $x^2+y^2-Ry=0$, where $R>0$. Find the surface area of $S$.

**Solution:** We first set $f = x^2+y^2+z^2-R^2$. Then $\dfrac{|\nabla f|}{|\nabla f \cdot p|} = \dfrac{R}{z}$.

$$\begin{aligned}
\text{Area}(S) &= 2\int_0^R \int_0^{\sqrt{Ry-y^2}} \frac{R}{z}\, dxdy \\
&= 2R \int_0^R \int_0^{\sqrt{Ry-y^2}} \frac{1}{\sqrt{R^2-x^2-y^2}}\, dxdy \\
&= 2R \int_0^{\pi/2} \int_0^{R\sin\theta} \frac{r\, drd\theta}{\sqrt{R^2-r^2}} \\
&= 2R \int_0^{\pi/2} \left( -\sqrt{R^2-r^2}\Big|_0^{R\sin\theta} \right) d\theta \\
&= 2R \int_0^{\pi/2} (R - R\cos\theta)\, d\theta \\
&= 2R^2 \left( \theta - \sin\theta \Big|_0^{\pi/2} \right) \\
&= R^2(\pi - 2).
\end{aligned}$$

If you choose to integrate from $\theta = 0$ to $\theta = \pi$ from the beginning, then you should note that $\sqrt{\cos^2\theta} = |\cos\theta|$ which is $\cos\theta$ when $0 \leq \theta \leq \pi/2$ and $-\cos\theta$ when $\pi/2 \leq \theta \leq \pi$.