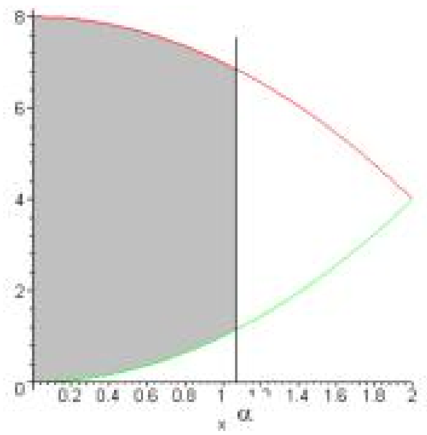
NAME: STUDENT NO:

Q-4) For $0 < \alpha < 2$, define

$$F(\alpha) = \int_0^{\alpha^2} \int_0^{\sqrt{y}} f \, dxdy + \int_{\alpha^2}^{8-\alpha^2} \int_0^{\alpha} f \, dxdy + \int_{8-\alpha^2}^{8} \int_0^{\sqrt{8-y}} f \, dxdy$$

where $f = \dfrac{y \sin x}{4 - x^2}\cdot$ Evaluate $F(\pi/3)$.

Solution: The region of integration is the shaded region of the following figure.



Changing the order of integration on this region we find

$$F(\alpha) = \int_0^{\alpha} \int_{x^2}^{8-x^2} \frac{\sin x}{4 - x^2} \, y \, dydx = 8 \int_0^{\alpha} \sin x \, dx = 8(1 - \cos\alpha).$$

Now we easily calculate $F(\pi/3) = 4$.