NAME: STUDENT NO:

**Q-5)** Let $\mathbf{F} = -y\ln(e+z^2)\,\mathbf{i} + x(x^2+y^2)\sec z\,\mathbf{j} + (x-y+z)\ln(4+x^4+y^4-z)\,\mathbf{k}$ be a field defined on the hemisphere $S$ given by $x^2+y^2+z^2=1$, $z \geq 0$. Calculate explicitly

$$\int\!\!\int_S \text{curl } \mathbf{F}\cdot\mathbf{n}\,d\sigma.$$

**Solution:** We use Stokes' theorem which says

$$\int\!\!\int_S \text{curl } \mathbf{F}\cdot\mathbf{n}\,d\sigma = \oint_C \mathbf{F}\cdot d\mathbf{r},$$

where $C$ is the boundary of $S$. In our case $C$ corresponds to $z=0$, but then $\mathbf{F} = -y\,\mathbf{i} + x(x^2+y^2)\,\mathbf{j} + (x-y)\ln(4+x^4+y^4)\,\mathbf{k}$.

A parametrization for the boundary is $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + 0\,\mathbf{k}$, for $0 \leq t \leq 2\pi$. Then $d\mathbf{r}(t) = (-\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + 0\,\mathbf{k})\,dt$. Putting in the parametrization of the boundary into $\mathbf{F}$ and calculating $\mathbf{F}\cdot d\mathbf{r}$ gives

$$\mathbf{F}\cdot d\mathbf{r} = (-\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \textit{something}\,\mathbf{k})\cdot(-\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + 0\,\mathbf{k})\,dt = dt.$$

Hence the right hand side integral gives $2\pi$ as the final answer.