Math 102 Calculus II – Homework I

Due on July 6, 2007 Friday 17:00

Q-1) For $0 < \alpha < 2$, define

$$F(\alpha) = \int_0^{\alpha^2}\int_0^{\sqrt{y}} f\, dxdy + \int_{\alpha^2}^{8-\alpha^2}\int_0^{\alpha} f\, dxdy + \int_{8-\alpha^2}^{8}\int_0^{\sqrt{8-y}} f\, dxdy$$

where $f = \dfrac{y\sin x}{4-x^2}\cdot$ Evaluate $F(\alpha)$ explicitly in terms of α.

Q-2) Find the area of the region common to the cardioids $r = 1+\sin\theta$ and $r = 1+\cos\theta$.

Q-3) Let $F(a)$ denote the volume of the region common to the cylinders $x^2+y^2 = 1$ and $x^2+z^2 = a^2$, where $a \geq 1$. Write the integral expression for $F(a)$. Evaluate $F(1)$ explicitly. Using a computer software find a such that $F(a) = 2F(1)$.

Q-4) Find the volume of the region bounded from above by $x^2+y^2+z^2 = 4$, from below by $z = 1$ and from the sides by $x^2+y^2-2y = 0$.

Q-5) For $n \geq 2$, let V_n denote the *volume* of the region

$$\{(x_1,\dots,x_n) \in \mathbb{R}^n \mid x_1^2+\cdots+x_n^2 \leq 1\}.$$

For example $V_2 = \pi$ and $V_3 = 4\pi/3$. Find V_4 and V_5.

---

sertoz@bilkent.edu.tr