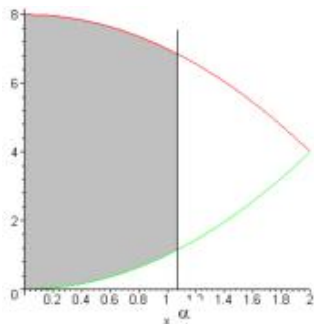


Q-1) For $0 < \alpha < 2$, define

$$F(\alpha) = \int_0^{\alpha^2} \int_0^{\sqrt{y}} f \, dx dy + \int_{\alpha^2}^{8-\alpha^2} \int_0^{\alpha} f \, dx dy + \int_{8-\alpha^2}^8 \int_0^{\sqrt{8-y}} f \, dx dy$$

where $f = \frac{y \sin x}{4-x^2}$. Evaluate $F(\alpha)$ explicitly in terms of α .

Solution: The region of integration is the shaded region of the following figure.



Changing the order of integration on this region we find

$$F(\alpha) = \int_0^{\alpha} \int_{x^2}^{8-x^2} \frac{\sin x}{4-x^2} y \, dy dx = 8 \int_0^{\alpha} \sin x \, dx = 8(1 - \cos \alpha).$$