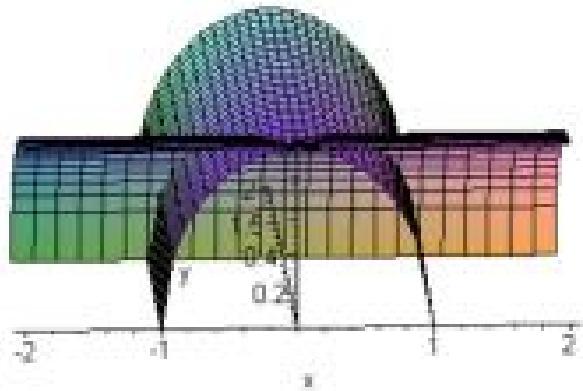


Q-3) Let $F(a)$ denote the volume of the region common to the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = a^2$, where $a \geq 1$. Write the integral expression for $F(a)$. Evaluate $F(1)$ explicitly. Using a computer software find a such that $F(a) = 2F(1)$.

Solution:

Two cylinders of the same radii in general intersect as follows:



In our case we find

$$F(a) = 8 \int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{a^2 - x^2} \, dy dx = 8 \int_0^1 \sqrt{(1-x^2)(a^2 - x^2)} \, dx.$$

We easily find $F(1) = \frac{16}{3}$.

It turns out that if $a = \sqrt{3.143} \approx 1.77$, then $F(a) \approx 2F(1)$.