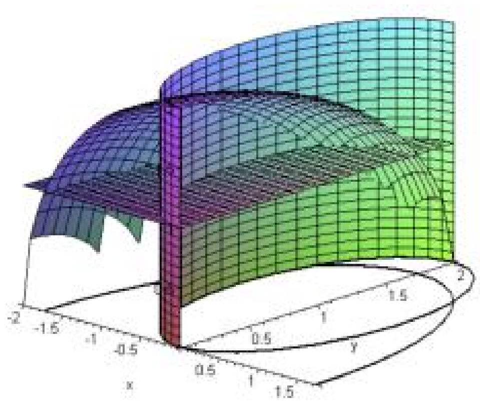
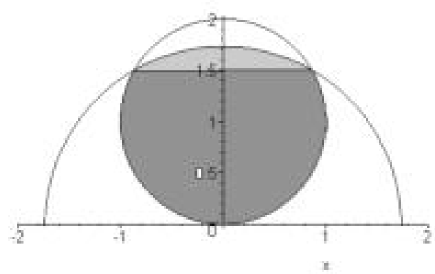
Q-4) Find the volume of the region bounded from above by $x^2+y^2+z^2=4$, from below by $z=1$ and from the sides by $x^2+y^2-2y=0$.

Solution:

The region is as follows.



The shadow of the $z=1$ base of the region in the xy-plane is as follows.



Note that the semicircle here is $x^2+y^2=3$ obtained by putting $z=1$ in the sphere equation.

Using the symmetry we set up the volume integral as

$$2\int_0^{3/2}\int_0^{\sqrt{2y-y^2}}\int_1^{\sqrt{4-x^2-y^2}} dzdxdy + 2\int_{3/2}^{\sqrt{3}}\int_0^{\sqrt{3-y^2}}\int_1^{\sqrt{4-x^2-y^2}} dzdxdy.$$

Changing to cylindrical coordinates

$$2\int_0^{\pi/3}\int_0^{2\sin\theta}\int_1^{\sqrt{4-r^2}} rdzdrd\theta + 2\int_{\pi/3}^{\pi/2}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} rdzdrd\theta$$

and evaluating we find the first integral as $2\left(\dfrac{5\pi}{9}-\dfrac{3\sqrt{3}}{4}\right)$, and the second integral as $2\left(\dfrac{5\pi}{36}\right)$. Hence the volume is $\dfrac{25\pi}{18}-\dfrac{3\sqrt{3}}{2}\approx 1.76$.