

Q-1) Let $\mathbf{F} = x^2 \mathbf{i} + z \mathbf{j} + yz \mathbf{k}$, and C the curve parametrized as $\mathbf{r} = \cos t \mathbf{i} + \sin t \mathbf{j} + t^2 \mathbf{k}$ for $0 \leq t \leq \pi$. Evaluate the work integral

$$\int \mathbf{F} \cdot \mathbf{T} \, ds$$

on the curve C .

Solution:

$$\begin{aligned} \int \mathbf{F} \cdot \mathbf{T} \, ds &= \int \mathbf{F} \cdot d\mathbf{r} \\ &= \int (x^2, z, yz) \cdot (dx, dy, dz) \\ &= \int_0^\pi (\cos^2 t, t^2, t^2 \sin t) \cdot (-\sin t, \cos t, 2t) \, dt \\ &= \int_0^\pi (-\sin t \cos^2 t + t^2 \cos t + 2t^3 \sin t) \, dt \\ &= \left[\frac{1}{3} \cos^3 t + 7t^2 \sin t - 14 \sin t + 14t \cos t - 2t^3 \cos t \right]_0^\pi \\ &= -\frac{2}{3} + 2\pi^3 - 14\pi \approx 17.36. \end{aligned}$$