

Q-2) Find a potential function $f(x, y, z)$ for the field $\mathbf{F} = \ln x \mathbf{i} + \cos(y+z) \mathbf{j} + (z + \cos(y+z)) \mathbf{k}$ such that $f(1, \frac{\pi}{2} - 1, 1) = -1$.

Solution: $f_x(x, y, z) = \ln x$, so $f = x \ln x - x + \phi(y, z)$.

$f_y(x, y, z) = \phi_y(y, z) = \cos(y+z)$, so $\phi(y, z) = \sin(y+z) + \alpha(z)$, and this gives $f(x, y, z) = x \ln x - x + \sin(y+z) + \alpha(z)$.

$f_z(x, y, z) = \cos(y+z) + \alpha'(z) = \cos(y+z) + z$, so $\alpha(z) = \frac{1}{2}z^2 + C$, and this gives $f(x, y, z) = x \ln x - x + \sin(y+z) + \frac{1}{2}z^2 + C$.

And finally $f(1, \frac{\pi}{2} - 1, 1) = -1$ determines C as $-\frac{3}{2}$.