

Q-3) Find the area enclosed by the simple curve C parametrized as $\mathbf{r}(t) = t^4 \mathbf{i} + (t - t^3) \mathbf{j}$ for $-1 \leq t \leq 1$. May I remind you that area is and should be a non-negative number. If you find a negative number, you owe an explanation!

Solution: Let R be the region bounded by the curve C . Observe that the parametrization of C traverses the boundary of R in clockwise direction which is the reverse direction for the application of Green's theorem. Therefore we need the following minus sign in the formula

$$\begin{aligned} \text{Area}(R) &= -\frac{1}{2} \int_C xdy - ydx \\ &= -\frac{1}{2} \int_{-1}^1 [(t^4)(1 - 3t^2) - (t - t^3)(4t^3)] dt \\ &= -\frac{1}{2} \int_{-1}^1 (-3t^4 + t^6) dt \\ &= -\frac{1}{2} \frac{1 - 32}{35} = \frac{16}{35}. \end{aligned}$$

