Q-5) Let $\mathbf{F} = x\ln(1+z^2)\ \mathbf{i} + y\tan z\cos x\ \mathbf{j} + z\ln(4+x^4+y^4)\ \mathbf{k}$ be a field defined on the hemisphere S given by $x^2+y^2+z^2=1$, $z \geq 0$. Calculate explicitly

$$\int\int_S \text{curl } \mathbf{F} \cdot \mathbf{n}\ d\sigma.$$

Solution: We use Stokes' theorem which says

$$\int\int_S \text{curl } \mathbf{F} \cdot \mathbf{n}\ d\sigma = \oint_C \mathbf{F} \cdot d\mathbf{r},$$

where C is the boundary of S. In our case C corresponds to $z = 0$, but then $\mathbf{F} = 0$, so the required integral is zero.