STUDENT NO:

Q-4) Find all values of x for which the power series $\sum_{n=2}^{\infty} \frac{7^n}{n \ln n} x^n$ converges.

Solution:

Use ratio test for the absolute values. $\frac{|a_{n+1}|}{|a_n|} = \frac{n \ln n}{(n+1) \ln(n+1)} \, 7|x| \to 7|x| \text{ as } n \to \infty.$

For absolute convergence we must have |x| < 1/7.

When x = 1/7, $a_n = 1/(n \ln n)$, and the series diverges by the integral test.

When x = -1/7, $a_n = (-1)^n/(n \ln n)$, and the series converges by the alternating series test.

The series converges only for $x \in [-1/7, 1/7)$.