NAME:

Q-5) Find
$$\lim_{x \to 0} \frac{x \cos x - \sin x + \frac{x^3}{3}}{x^3 (\sin x^2) \ln(2+x)}$$
.

Solution:

 $\ln(2+x)=\ln 2$ when x=0, so it does not contribute to the indeterminacy of the above limit.

$$x\cos x - \sin x + \frac{x^3}{3} = \frac{1}{30}x^5 - x^7(*),$$
$$x^3(\sin x^2) = x^5 - x^9(*).$$

Cancelling out x^5 and substituting x = 0, we find the limit as $\frac{1}{30 \ln 2}$.

Note: If $-\frac{x^3}{3}$ is used instead of $+\frac{x^3}{3}$, then the limit is clearly $-\infty$. If you solved it that way, your answer is correct and is certainly acceptable.