Date: June 14, 2007, Thursday NAME:..
Time: 9:30-11:30
Ali Sinan Sertöz STUDENT NO:..

Math 102 Calculus II – Midterm Exam I

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Please do not write anything inside the above boxes!

PLEASE READ:
Check that there are 5 questions on your exam booklet. Write your name on the top of every page. Show your work in reasonable detail. A correct answer without proper reasoning may not get any credit.

Q-1) Find $\lim_{n\to\infty} a_n$, where $a_n = \left(1+\dfrac{2}{3n}\right)^{4n}$, $n = 1, 2, \ldots$.

Solution:

$$a_n = \left[\left(1+\frac{2/3}{n}\right)^n\right]^4 \longrightarrow \left[e^{2/3}\right]^4 = e^{8/3} \text{ as } n \to \infty.$$

Or you can consider $\ln a_n = \dfrac{\ln(1+2/3n)}{1/4n}$ and use L'Hopital's rule as $n \to \infty$.

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Q-2) Check the following series for converge:

(i) $\displaystyle\sum_{n=2}^{\infty} \frac{3n+7}{(8n^2+11n+2007)(\ln n)^2}$ **(ii)** $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n+1)}{e^n \; n!}$.

Solution:

(i): Limit compare with $\sum \frac{1}{n(\ln n)^2}$ which converges by the integral test, to conclude that the given series converges.

(ii): Use ratio test, $\frac{a_{n+1}}{a_n} = \frac{2n+3}{e\,(n+1)} \to \frac{2}{e} < 1$ as $n \to \infty$, to conclude that the series converges.

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Q-3) Let $a_{2n-1} = \dfrac{1}{5^n}$, $a_{2n} = -\dfrac{1}{8^n}$ for $n = 1, 2, \ldots$. Show that the series $\sum_{n=1}^{\infty} a_n$ converges absolutely. Find its sum. Justify your calculations.

Solution:

Use the root test for $|a_N|$ to find that the series converges absolutely. Then any rearrangement of the terms will converge to the same sum. First add up the positive terms, then the negative terms as geometric series to find the sum as $\dfrac{3}{28}$.

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Q-4) Find all values of x for which the power series $\displaystyle\sum_{n=2}^{\infty} \frac{7^n}{n \ln n}\, x^n$ converges.

Solution:

Use ratio test for the absolute values. $\dfrac{|a_{n+1}|}{|a_n|} = \dfrac{n \ln n}{(n+1)\ln(n+1)}\, 7|x| \to 7|x|$ as $n \to \infty$.

For absolute convergence we must have $|x| < 1/7$.

When $x = 1/7$, $a_n = 1/(n \ln n)$, and the series diverges by the integral test.

When $x = -1/7$, $a_n = (-1)^n/(n \ln n)$, and the series converges by the alternating series test.

The series converges only for $x \in [-1/7, 1/7)$.

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Q-5) Find $\lim_{x \to 0} \dfrac{x \cos x - \sin x + \frac{x^3}{3}}{x^3 (\sin x^2) \ln(2+x)}$.

Solution:

$\ln(2+x) = \ln 2$ when $x = 0$, so it does not contribute to the indeterminacy of the above limit.

$x \cos x - \sin x + \dfrac{x^3}{3} = \dfrac{1}{30}x^5 - x^7(*)$,

$x^3(\sin x^2) = x^5 - x^9(*)$.

Cancelling out x^5 and substituting $x = 0$, we find the limit as $\dfrac{1}{30 \ln 2}$.

Note: If $-\dfrac{x^3}{3}$ is used instead of $+\dfrac{x^3}{3}$, then the limit is clearly $-\infty$. If you solved it that way, your answer is correct and is certainly acceptable.