NAME: STUDENT NO:

Q-2) Let $f(x,y,z) = x^2 + xy + yz$, $x(s,t) = s^2 - st + 2t + 2s - 2$, $y(s,t) = t^2 + s^2t + s - 8$, $z(s,t) = s^2t - 1$. Find the directional derivative of $h(s,t) = f(x(s,t), y(s,t), z(s,t))$ at the point $(s,t) = (1,2)$ in the direction of the vector $\vec{u} = (3,4)$.

Solution: $h_s = \nabla f \cdot (x_s, y_s, z_s)$, $h_t = \nabla f \cdot (x_t, y_t, z_t)$.
$\nabla f = (2x + y, x + z, y)$.
$x(1,2) = 3$, $y(1,2) = -1$, $z(1,2) = 1$, so $\nabla f(3,-1,1) = (5,4,-1)$.
$x_s = 2s - t + 2 = 2$ at $(s,t) = (1,2)$.
$y_s = 2st + 1 = 5$ at $(s,t) = (1,2)$.
$z_s = 2st = 4$ at $(s,t) = (1,2)$.

$h_s = (5,4,-1) \cdot (2,5,4) = 26$.

$x_t = -s + 2 = 1$ at $(s,t) = (1,2)$.
$y_t = 2t + s^2 = 5$ at $(s,t) = (1,2)$.
$z_t = s^2 = 1$ at $(s,t) = (1,2)$.

$h_t = (5,4,-1) \cdot (1,5,1) = 24$.

$\nabla h(1,2) = (26,24)$, $\nabla h(1,2) \cdot (3,4) = (26,24) \cdot (3,4) = 174$, $|\vec{u}| = 5$

Finally we have $D_{\vec{u}} h(1,2) = \frac{1}{5} \nabla h(1,2) \cdot (3,4) = \frac{174}{5}$.