Q-4) Find and classify all the critical points of the function $f(x, y) = x^3 - 2xy^2 - x + 2y^2$.

Solution:
$f_x = 3x^2 - 2y^2 - 1 = 0$
$f_y = -4xy + 4y = 4y(1 - x) = 0$

Case 1: $y = 0$. From $f_x = 0$ we get $x = \pm 1/\sqrt{3}$. The critical points in this case are $(\pm 1/\sqrt{3}, 0)$.

Case 2: $y \neq 0$. Then $x = 1$ and from $f_x = 0$ we get $y = \pm 1$. The critical points of this case are $(1, \pm 1)$.

$f_{xx} = 6x$, $f_{yy} = 4(1 - x)$, $f_{xy} = -4y$, $\Delta = 8[3x(1 - x) - 2y^2]$.

At $(1/\sqrt{3}, 0)$, $\Delta > 0$, $f_{xx} > 0$, so this is a local minimum point.

At the other critical points $\Delta < 0$, so they are all saddle points.