NAME: STUDENT NO:

Q-5) Find the minimum and the maximum values of the function $f(x,y,z) = x^2 - y^3 + 3z^2$ on the sphere $x^2 + y^2 + z^2 = 25$.

Solution: Let $g(x,y,z) = x^2+y^2+z^2-25$. $\nabla f = \lambda \nabla g$ gives $(2x, -3y^2, 6z) = \lambda(x,y,z)$, or:
$x(\lambda - 2) = 0$
$y(\lambda + 3y) = 0$
$z(\lambda - 6) = 0.$

Case 1: $x = 0$.
Case 1.1: $y = 0$. Then $g(0,0,z) = 0$ gives $z = \pm 5$, $\boxed{f(0,0,\pm 5) = 75}$.
Case 1.2: $y \neq 0$. Then $y = -\lambda/3$.
Case 1.2.1: $z = 0$. From $g(0,y,0) = 0$ we get $y = \pm 5$. $\boxed{f(0,5,0) = -125}$, $\boxed{f(0,-5,0) = 125}$.
Case 1.2.2: $z \neq 0$. Then $\lambda = 6$ and hence $y = -2$. From $g(0,-2,z) = 0$ we get $z = \pm\sqrt{21}$. $\boxed{f(0,-2,\pm\sqrt{21}) = 71}$.
Case 2: $x \neq 0$. Then $\lambda = 2$ and this forces $z = 0$.
Case 2.1: $y = 0$. Then $g(x,0,0) = 0$ gives $x = \pm\sqrt{5}$. $\boxed{f(\pm\sqrt{5},0,0) = 25}$.
Case 2.2: $y \neq 0$. Then $y = -\lambda/3 = -2/3$. $g(x,-2/3,0) = 0$ gives $x = \pm\sqrt{25 - 4/9}$.
$\boxed{f(\pm\sqrt{25-4/9}, -2/3, 0) = 25 - 4/27}$.

Thus we see that the maximal value is 125 and the minimal value is -125.