

Q-2) Find the volume of the region bounded by the paraboloid $x^2 + y^2 + z = 4$ and the cylinder $x^2 - 2y + y^2 = 0$ above the xy -plane.

Hint: $\int \sin^4 t \, dt = \frac{3t}{8} - \sin(2t) \left(\frac{3}{16} + \frac{1}{8} \sin^2 t \right) + C.$

Solution:

$$\begin{aligned} \text{Volume} &= \int_0^2 \int_{-\sqrt{2y-y^2}}^{\sqrt{2y-y^2}} \int_0^{4-x^2-y^2} dz \, dx \, dy \\ &= \int_0^\pi \int_0^{2\sin\theta} (4-r^2)r \, dr \, d\theta \\ &= \int_0^\pi \left[2r^2 - \frac{1}{4}r^4 \right]_0^{2\sin\theta} d\theta \\ &= \int_0^\pi (8\sin^2\theta - 4\sin^4\theta) \, d\theta \\ &= \left[\sin^3\theta \cos\theta - \frac{5}{2}\sin\theta \cos\theta + \frac{5t}{2} \right]_0^\pi \\ &= \frac{5\pi}{2}. \end{aligned}$$