Q-3) Let C be the curve parameterized as $r(\theta) = \left(\frac{\theta}{8},\ \sin^2\theta,\ 1-\cos^4\theta\right)$, with $0 \le \theta \le 2\pi$. Calculate

$$\int_C \mathbf{F}\cdot\mathbf{T}\,ds,$$

where $\mathbf{F} = (-\tan(x+y^2+z^3),\ -2y\tan(x+y^2+z^3),\ -3z^2\tan(x+y^2+z^3))$.

Solution: In the last homework we showed that this is a conservative field with potential function $f = \ln\cos(x+y^2+z^3)+C$. This gives

$$\int_C \mathbf{F}\cdot\mathbf{T}\,ds = f(r(2\pi)) - f(r(0)) = f(\pi/4,0,0) - f(0,0,0) = -\frac{1}{2}\ln 2.$$