Q-4) Find the surface area of the piece of the paraboloid $x^2 + y^2 + z = 4$ with $z \geq 0$.

Solution: If $f = x^2 + y^2 + z - 4$ and D is the projection of the paraboloid S to xy-plane, then the surface area is given by

$$
\begin{aligned}
\int_S d\sigma &= \int_D \frac{|\nabla f|}{|\nabla f \cdot \mathbf{k}|} \, dxdy \\
&= \int_0^{2\pi} \int_0^2 \sqrt{1 + 4r^2} \, r \, drd\theta \\
&= (2\pi) \left[\frac{1}{12}(1 + 4r^2)^{3/2} \Big|_0^2 \right] \\
&= \frac{\pi}{6}(17\sqrt{17} - 1).
\end{aligned}
$$