Q-1) Let $f(x,y) = y^2 \sin\left(\dfrac{xy^2\pi}{6}\right)$. Evaluate the following integral:

$$\mathbf{I} = \int_{1/2}^{1}\int_{1/x}^{2} f(x,y)\,dydx + \int_{1}^{3}\int_{1}^{2} f(x,y)\,dydx + \int_{3}^{6}\int_{1}^{6/x} f(x,y)\,dydx.$$

Solution: Sketch the region, reverse the order of integration to get

$$\begin{aligned}
\mathbf{I} &= \int_{1}^{2}\int_{1/y}^{6/y} y^2 \sin\left(\frac{xy^2\pi}{6}\right)\,dxdy \\
&= \int_{1}^{2}\left(-\frac{6\cos\left(xy^2\pi/6\right)}{\pi}\Bigg|_{1/y}^{6/y}\right) \\
&= \frac{6}{\pi}\int_{1}^{2}\left(\cos(y\pi/6) - \cos(y\pi)\right)\,dy \\
&= \frac{6}{\pi^2}\left(\left(6\sin(y\pi/6) - \sin(y\pi)\right)\Big|_{1}^{2}\right) \\
&= \frac{18(\sqrt{3}-1)}{\pi^2} \approx 1.33.
\end{aligned}$$