

Q-2) Find the area, in the first quadrant, that is both inside the circle $r = \sqrt{2}$ and the lemniscate $r = \sqrt{4 \cos 2\theta}$.

Solution: These curves intersect when $\theta = \pi/6$. From $\theta = 0$ to $\theta = \pi/6$, it is the area of the circle, and from $\theta = \pi/6$ to $\theta = \pi/4$, it is the area of the lemniscate which constitute the common area. Hence

$$\text{Area} = \frac{\pi}{6} + \int_{\pi/6}^{\pi/4} \int_0^{\sqrt{4 \cos 2\theta}} r \, dr \, d\theta = \frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2} \approx 0.657.$$