Q-3) Set up an integral to evaluate the volume of the region common to two right circular cylinders, of radii a and b where a > b > 0, intersecting orthogonally along their central axes.

Solution: Assume that the cylinders are $x^2 + y^2 = a^2$ and $y^2 + z^2 = b^2$. Then one eighth of the volume is given by the integral

$$\int_0^b \int_0^{\sqrt{b^2 - y^2}} \int_0^{\sqrt{a^2 - y^2}} dx dz dy = \int_0^b \sqrt{(a^2 - y^2)(b^2 - y^2)} dy.$$