Q-4) Find the volume of the region bounded from above by $x^2 + y^2 + z^2 = 4$, from below by $z = 1$, and from the sides by $x^2 + y^2 - 2y = 0$.

Solution: If you plot the region carefully, you will see that the volume is given by

$$2\int_0^{3/2}\int_0^{\sqrt{2y-y^2}}\int_1^{\sqrt{4-x^2-y^2}} dzdxdy + 2\int_{3/2}^{\sqrt{3}}\int_0^{\sqrt{3-y^2}}\int_1^{\sqrt{4-x^2-y^2}} dzdxdy.$$

After changing to cylindrical coordinates and evaluating these integrals you will find that their values are

$$2\left(\frac{5\pi}{9} - \frac{3\sqrt{3}}{4}\right) + 2\left(\frac{5\pi}{36}\right) = \frac{25\pi - 27\sqrt{3}}{18} \approx 1.765.$$