Q-3) Find the area of the surface S cut from the cone $z^2 = 4x^2 + 4y^2$, $z \geq 0$, by the cylinder $x^2 + y^2 = 2x$.

Solution: Let $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 2x \}$, and let $f = 4x^2 + 4y^2 - z^2$. Then the surface is given by $f = 0$ over D.

$d\sigma = \dfrac{|\nabla f|}{|\nabla f \cdot \mathbf{k}|} dA = \sqrt{5} dA$. Thus

$$\text{Surface area} = \int_S d\sigma = \int_D \frac{|\nabla f|}{|\nabla f \cdot \mathbf{k}|} dA = \sqrt{5} \int_D dA = \sqrt{5}\,\pi.$$