**Q-5)** Solve the very last problem of the book, exercise 21 on page 1228:
Show that the volume of a region $D$ in space enclosed by the oriented surface $S$ with outward normal $\mathbf{n}$ satisfies the identity

$$V = \frac{1}{3} \int\int_S \mathbf{r} \cdot \mathbf{n} \, d\sigma,$$

where $\mathbf{r}$ is the position vector of the point $(x, y, z)$ in $D$.

**Solution:** Taking $\mathbf{r}$ as the vector field $\mathbf{F}$ of the divergence theorem, we find that

$$\int\int_S \mathbf{r} \cdot \mathbf{n} \, d\sigma = \int\int\int_D \nabla \cdot \mathbf{r} \, dV = \int\int\int_D 3 \, dV = 3V,$$

verifying the required equality.