Q-3) Find the sum

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)(n+3)}$$

Solution:

$$\frac{n}{(n+1)(n+2)(n+3)} = \frac{-1/2}{n+1} + \frac{2}{n+2} + \frac{-3/2}{n+3}.$$

Adding these from $n = 1$ to $n = k$ we find

$$s_k = \frac{1}{4} - \frac{3+2k}{2(2+k)(3+k)}$$

Hence the sum is $\lim_{k\to\infty} s_k = \frac{1}{4}$.