Q-4) Find the radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$. *(10 points)*

Check the convergence of the series at the end points. *(10 points)*

Hint: $f(x) = (1 + 1/x)^x$ *is an increasing function for* $x > 1$.

Solution:

Let $a_n = \frac{n!}{n^n} x^n$ and use ratio test for the absolute values. $\frac{|a_{n+1}|}{|a_n|} = \frac{|x|}{(1 + 1/n)^n} \to |x|/e$ as $n \to \infty$.

For absolute convergence we must have $|x| < e$. So the radius of convergence is e.

When $x = \pm e$, we have $\frac{|a_{n+1}|}{|a_n|} = \frac{e}{(1 + 1/n)^n} > 1$, using the hint. Hence $a_n > a_1$ for all n and the general term a_n does not converge to zero as n goes to infinity, and the series diverges at the end points.