Q-5) Find the values of c and d (*5 points each*) such that the following limit exists and is finite. For those values of c and d find the limit. (*10 points*)

$$\lim_{x \to 0} \left(\frac{\cos(x^2)}{x^8} + \frac{c \sin x}{x^2} + \frac{d + cx^4}{x^8} - \frac{1}{2x} + \frac{d}{c} \right)$$

Solution:

The expression in the limit has the Taylor expansion

$$\left((1+d)\, x^{-8} + \left(-\frac{1}{2} + c \right) x^{-4} + \left(-\frac{1}{2} + c \right) x^{-1} + \frac{d}{c} + \frac{1}{24} - \frac{1}{6} cx + \frac{1}{120} cx^3 + \cdots \right)$$

For the limit to exist and be finite we need to have $d = -1$ and $c = 1/2$. And in that case the limit is $-\dfrac{47}{24}$.