Date: June 19, 2008, Thursday NAME:..
Time: 9:30-11:30
Ali Sinan Sertöz STUDENT NO:..

Math 102 Calculus II – Midterm Exam I

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Please do not write anything inside the above boxes!

PLEASE READ:
Check that there are 5 questions on your exam booklet. Write your name on the top of every page. Show your work in reasonable detail. A correct answer without proper reasoning may not get any credit.

Q-1) Find $\lim_{n\to\infty} a_n$, where $a_n = (\ln n)^{1/\ln n}$, $n = 2, 3, \ldots$.

Solution:

You can consider $\ln a_n = \dfrac{\ln(\ln n)}{\ln n}$ and use L'Hopital's rule as $n \to \infty$. This will give $\lim_{n\to\infty} a_n = 1$

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Q-2) Check the following series for converge:

$$\sum_{n=1}^{\infty} \frac{\ln n}{(19n^2 + 6n + 2008)}$$

Solution:

Limit compare with $\sum \frac{\ln n}{n^2}$ which converges by the integral test, to conclude that the given series converges.

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Q-3) Find the sum

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)(n+3)}$$

Solution:

$$\frac{n}{(n+1)(n+2)(n+3)} = \frac{-1/2}{n+1} + \frac{2}{n+2} + \frac{-3/2}{n+3}.$$

Adding these from $n = 1$ to $n = k$ we find

$$s_k = \frac{1}{4} - \frac{3+2k}{2(2+k)(3+k)}$$

Hence the sum is $\lim_{k\to\infty} s_k = \frac{1}{4}$.

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Q-4) Find the radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$. *(10 points)*

Check the convergence of the series at the end points. *(10 points)*

Hint: $f(x) = (1 + 1/x)^x$ is an increasing function for $x > 1$.

Solution:

Let $a_n = \frac{n!}{n^n} x^n$ and use ratio test for the absolute values. $\frac{|a_{n+1}|}{|a_n|} = \frac{|x|}{(1 + 1/n)^n} \to |x|/e$ as $n \to \infty$.

For absolute convergence we must have $|x| < e$. So the radius of convergence is e.

When $x = \pm e$, we have $\frac{|a_{n+1}|}{|a_n|} = \frac{e}{(1 + 1/n)^n} > 1$, using the hint. Hence $a_n > a_1$ for all n and the general term a_n does not converge to zero as n goes to infinity, and the series diverges at the end points.

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Q-5) Find the values of c and d *(5 points each)* such that the following limit exists and is finite. For those values of c and d find the limit. *(10 points)*

$$\lim_{x\to 0}\left(\frac{\cos(x^2)}{x^8}+\frac{c\,\sin x}{x^2}+\frac{d+cx^4}{x^8}-\frac{1}{2x}+\frac{d}{c}\right)$$

Solution:

The expression in the limit has the Taylor expansion

$$\left((1+d)\,x^{-8}+\left(-\frac{1}{2}+c\right)x^{-4}+\left(-\frac{1}{2}+c\right)x^{-1}+\frac{d}{c}+\frac{1}{24}-\frac{1}{6}\,cx+\frac{1}{120}\,cx^3+\cdots\right)$$

For the limit to exist and be finite we need to have $d=-1$ and $c=1/2$. And in that case the limit is $-\dfrac{47}{24}$.