

Q-1) Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y^2}{x^4 + x^2 y^2 + y^4}$.

Solution:

First observe that

$$\left| \frac{x^3 y^2}{x^4 + x^2 y^2 + y^4} \right| = \frac{(x^2 y^2) |x|}{x^4 + x^2 y^2 + y^4} \leq \frac{(x^4 + x^2 y^2 + y^4) |x|}{x^4 + x^2 y^2 + y^4} \leq |x|.$$

Then conclude by the sandwich theorem that the required limit is zero.