Q-4) Find and classify all the critical points of $f(x, y) = x^3 + y^2 + x^2y$.

Solution:

$f_x = 0$ and $f_y = 0$ give $(0, 0)$ and $(3, -9/2)$ as the critical points.

At $(0, 0)$ the discriminant is zero. But $f(x, 0) = x^3$ takes both positive and negative values in every neighborhood of the origin, so the origin is a saddle point.

At the other critical point the discriminant is negative so it is also a saddle point.