

Q-5) Find the points on the surface $z^3 + x^2y = 1$ closest to the origin.

It may be necessary to notice that $(108/31)^{2/3} > 22/10$, and that $(16)^{1/3} > 2$.

Solution:

Using the Lagrange multipliers method you will find that the points $(0, 0, 1)$, $(\pm 4^{1/3}/\sqrt{2}, 4^{1/3}/2, 0)$ and $(\pm c/\sqrt{2}, c/2, c/3)$, where $c^3 = 108/31$ are the critical points. A brief comparison shows that the second and third sets of points have a distance larger than 1 from the origin. So the point $(0, 0, 1)$ is the point on the surface closest to the origin.